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10MAT31

Third Semester B.E. Degree Examination, December 2012
Engineering Mathematics – III

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. Find the Fourier series of $f(x) = x - x^2$, $-\pi \leq x \leq \pi$. Hence deduce that
- $$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$
- (07 Marks)

Is the above deduced series convergent? (Answer in Yes or No)

- b. Define : i) Half range Fourier sine series of $f(x)$
 ii) Complex form of Fourier series of $f(x)$
 Find the half range cosine series of $f(x) = x$ in $0 < x < 2$. (07 Marks)

- c. Obtain a_0, a_1, b_1 in the Fourier expansion of y , using harmonic analysis for the data given.

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(06 Marks)

- 2 a. Find the Fourier transform of
- $$f(x) = \begin{cases} 1 - x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$$
- Hence evaluate $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx$ (07 Marks)

- b. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ (07 Marks)

- c. Find the Fourier cosine transform of
- $$f(x) = \begin{cases} 4x & , \text{ for } 0 < x < 1 \\ 4 - x & , \text{ for } 1 < x < 4 \\ 0 & , \text{ for } x > 4 \end{cases}$$
- (06 Marks)

- 3 a. i) Write down the two dimensional heat flow equation (p d e) in steady state (or two dimensional) Laplace's equation. Just mention.
 ii) Solve one dimensional heat equation by the method of separation of variables. (07 Marks)
- b. Using D'Alembert's method, solve one dimensional wave equation. (07 Marks)
- c. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form of $y = a \sin(\pi x/l)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is,

$$y(x, t) = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi ct}{l}\right)$$

Start the answer assuming the solution to be

$$y = (C_1 \cos(px) + C_2 \sin(px))(C_3 \cos(cpt) + C_4 \sin(cpt))$$

(06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

- 4 a. Fit a linear law, $P = mW + C$, using the data

P	12	15	21	25
W	50	70	100	120

(06 Marks)

- b. Find the best values of a and b by fitting the law $V = at^b$ using method of least squares for the data,

V (ft/min)	350	400	500	600
t (min)	61	26	7	26

Use base 10 for algorithm for computation.

(07 Marks)

- c. Using simplex method,

$$\text{Maximize } Z = 5x_1 + 3x_2$$

$$\text{Subject to, } x_1 + x_2 \leq 2 ; 5x_1 + 2x_2 \leq 10 ; 3x_1 + 8x_2 \leq 12 ; x_1, x_2 \geq 0.$$

(07 Marks)

PART – B

- 5 a. Use Newton-Raphson method, to find the real root of the equation $3x = (\cos x) + 1$.

Take $x_0 = 0.6$. Perform two iterations.

(06 Marks)

- b. Apply Gauss-Seidel iteration method to solve equations

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3z + 20z = 25$$

Assume initial approximation to be $x = y = z = 0$. Perform three iterations.

(07 Marks)

- c. Using Rayleigh's power method to find the largest eigen value and the corresponding eigen vector of the matrix.

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Take $[1 \ 0 \ 0]^T$ as the initial approximation. Perform four iterations.

(07 Marks)

- 6 a. Use appropriate interpolating formula to compute $y(82)$ and $y(98)$ for the data

x	80	85	90	95	100
y	5026	5674	6362	7088	7854

(07 Marks)

- b. i) For the points (x_0, y_0) (x_1, y_1) (x_2, y_2) mention Lagrange's interpolation formula.

- ii) If $f(1) = 4$, $f(3) = 32$, $f(4) = 55$, $f(6) = 119$; find interpolating polynomial by Newton's divided difference formula.

(07 Marks)

- c. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$, using

- i) Simpson's $1/3^{\text{rd}}$ rule ii) Simpson's $3/8^{\text{th}}$ rule iii) Weddle's rule, using

x	0	1	2	3	4	5	6
$f(x) = \frac{1}{1+x^2}$	1	0.5	0.2	0.4	0.0588	0.0385	0.027

(06 Marks)

- 7 a. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$ subject to $u(0, t), u(4, t) = 0$. $u_t(x, 0) = 0$ and $u(x, 0) = x(4 - x)$ by taking $h = 1, k = 0.5$ upto four steps. (07 Marks)
- b. Solve two dimensional Laplace equation at the pivotal or nodal points of the mesh shown in Fig.Q7(b). To find the initial values assume $u_4 = 0$. Perform three iterations including computation of initial values. (07 Marks)

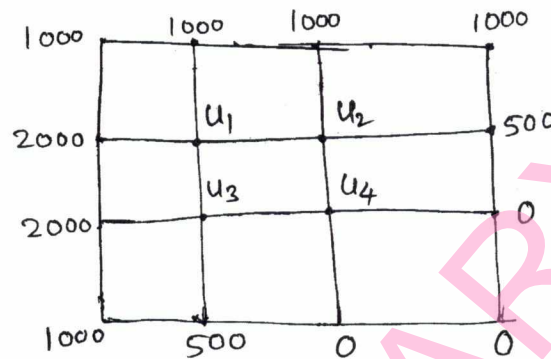


Fig.Q7(b)

- c. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to the conditions $u(x, 0) = \sin \pi x, 0 \leq x \leq 1$; $u(0, t) = u(1, t) = 0$. Carry out computations for two levels, taking $h = 1/3, k = 1/36$. (06 Marks)
- 8 a. Find the z-transform of $\frac{n}{3^n} + 2^n n^2 + 4 \cos(n\theta) + 4^n + 8$ (07 Marks)
- b. State and prove i) Initial value theorem ii) Final value theorem of z-transforms. (07 Marks)
- c. Using the z-transform solve $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0, u_1 = 1$. (06 Marks)

Third Semester B.E. Degree Examination, December 2012
Analog Electronic Circuits

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. Explain reverse recovery time of a semiconductor diode. (07 Marks)
 b. Explain avalanche breakdown and zener breakdown. (06 Marks)
 c. For the diode circuit shown in Fig.Q.1(c), calculate I_D , V_D and V_R , assume $V_T = 0.7V$. (07 Marks)

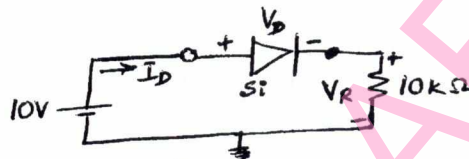


Fig.Q.1(c)

- 2 a. Explain with a neat diagram, fixed bias configuration to fix the operating point. (06 Marks)
 b. Derive the expression for stability factors for fixed-bias circuit, with respect to I_{CQ} , V_{BE} and β . (06 Marks)
 c. For the emitter bias circuit shown in Fig.Q.2(c), find the values of R_C , R_E and R_B using the following specifications $I_{C(sat)} = 10 \text{ mA}$, $I_{CQ} = 1/2 I_{C(sat)}$, $V_C = 20V$. Assume silicon transistor with $\beta = 100$. (08 Marks)

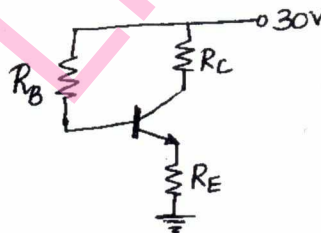


Fig.Q.2(c)

- 3 a. Obtain the expressions for voltage gain Z_{in} and Z_o of common-base configuration using AC equivalent circuit with r_e model. (07 Marks)
 b. Explain with a neat circuit diagram, Emitter follower configuration justify how. Voltage gain is approximated to unity. (07 Marks)
 c. For the circuit shown in Fig.Q.3(c) determine V_{CC} if $A_v = -160$ and $r_o = 100 \text{ k}\Omega$, take $\beta = 100$. (06 Marks)

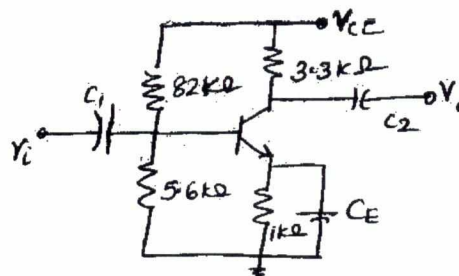


Fig.Q.3(c)

- 4 a. Describe the factors that affect the low frequency response of a BJT-CE amplifier. (10 Marks)
 b. For the common-base amplifier shown in Fig.Q.4(b) calculate r_e , R_i , A_v , C_s , C_c , over all lower cut-off frequency $\beta = 75$ and $r_0 = \infty$. (10 Marks)

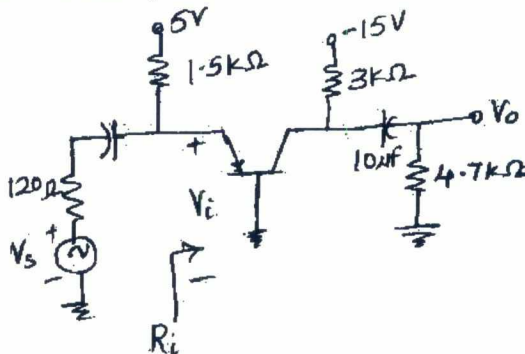


Fig.Q.4(b)

PART – B

- 5 a. With the help of a neat circuit diagram, explain the working of a Dalington Emitter-Follower and derive Z_i , A_i , A_v and Z_o . (10 Marks)
 b. List the general characteristics of a negative feedback amplifier and derive the expression for gain with negative feedback. (10 Marks)
- 6 a. Explain the operation of a class B push-pull power amplifier with the help of a neat circuit diagram and also draw the i/p and o/p waveforms of the class B power amplifier, justify elimination of even harmonic distortion. (10 Marks)
 b. A class B push-pull amplifier operating with $V_{CC} = 25V$ provides a 22V peak signal to an 8Ω load. Find. Peak load current, dc current drawn from the supply, input power, output power, circuit efficiency, power dissipation. (10 Marks)
- 7 a. Derive the expression for frequency of a Wein Bridge oscillator and explain the operation using a neat circuit diagram. (08 Marks)
 b. In a transistor Colpitts oscillator $C_1 = 1nF$ and $C_2 = 1000nF$. Find the value of L for a frequency of 100 kHz. (06 Marks)
 c. A crystal has the following parameter $L = 0.3344$, $C_M = 1pF$, $C = 0.065 pF$ and $R = 5.5 K\Omega$. Calculate the series resonant frequency, parallel resonant frequency and find the Q of the crystal. (06 Marks)
- 8 a. Explain the operation of JFET amplifier using fixed bias configuration. Draw the JFET small signal model and derive expressions for input impedances output impedance and voltage gain A_v . (10 Marks)
 b. For the JFET amplifier shown in Fig.Q.8(b). Calculate Z_i , Z_o and A_v . (10 Marks)

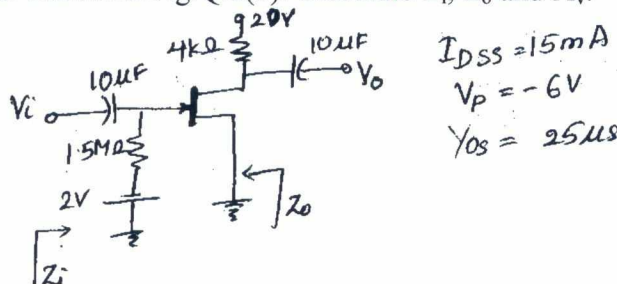


Fig.Q.8(b)

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Third Semester B.E. Degree Examination, December 2012

Logic Design

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. Define canonical Minterm form and canonical Maxterm form. (05 Marks)
- b. Design a three-input, one output minimal two-level gate combinational circuit which has an output equal to 1 when majority of its inputs are at logic 1 and has an output equal to 0 when majority of its inputs are at logic 0. (05 Marks)
- c. Minimize the following multiple output functions using K-MAP:

$$f_1 = \sum m(0, 2, 6, 10, 11, 12, 13) + d(3, 4, 5, 14, 15)$$

$$f_2 = \sum m(1, 2, 6, 7, 8, 13, 14, 15) + d(3, 5, 12)$$
 (10 Marks)
- 2 a. Use a K-Map to simplify that following functions:
 - i) $f(A, B, C, D) = (A + B + \bar{C})(\bar{B} + \bar{D})(\bar{A} + C)(B + C)$
 - ii) $f(A, B, C, D) = \pi(1, 2, 4, 5, 7, 8, 10, 11, 13, 14)$
 (10 Marks)
- b. Find all the prime implicants of the function

$$f(a, b, c, d) = \sum(7, 9, 12, 13, 14, 15) + \sum d(4, 11)$$
 Using Quine Mc Clusky algorithm. (10 Marks)
- 3 a. Reduce the given function using MEV technique:
 - i) $f = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + ABCE + ABC\bar{E} + d(\bar{A}\bar{B}CD + \bar{A}\bar{B}CE)$
 - ii) $f = m_0 + m_1F + m_2 + m_4F + m_6(E + \bar{E}) + m_7F + m_{10}E + m_{12} + m_{15}F + d(m_5F + m_9\bar{F} + m_{11}\bar{E} + m_8E)$
 (10 Marks)
- b. Write the condensed truth table for a 4 to 2 line priority encoder with a valid output where the highest priority is given to the highest bit position or input with highest index and obtain the minimal sum expressions for the outputs. (06 Marks)
- c. Describe general working principle of decoder. (04 Marks)
- 4 a. Explain the working principle of four-bit parallel fast look ahead carry adder. (10 Marks)
- b. Design a comparator to check if two n-bit numbers are equal. Configure this using cascaded stages of 1-bit comparators. (10 Marks)

PART – B

- 5 a. With a neat diagram, explain the working of Master-Slave JK flip-flop along with waveforms. (10 Marks)
- b. Explain switch debouncer using SR latch with waveforms. (10 Marks)
- 6 a. Explain universal shift register with the help of logic diagram, mode control table. (10 Marks)
- b. Design and implement a divide-by-10 asynchronous counter using T FFS. (10 Marks)

- 7 a. Design and implement a synchronous BCD counter using J-K FFS. (10 Marks)
 b. A sequential circuit has one input and one output state diagram is as shown in Fig.Q7(b). Design the sequential circuit with J-K flip-flop.

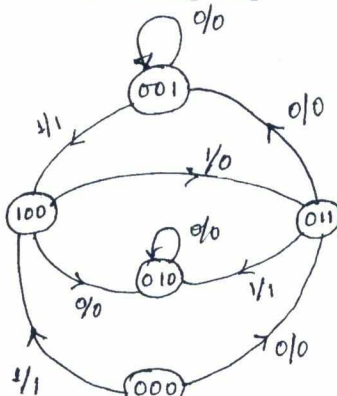


Fig.Q7(b)

(10 Marks)

- 8 a. Design a sequence detector for the following sequence 1, 0, 1, 1, 1 with overlap. Write the state diagram and logic diagram. (10 Marks)
 b. A sequential circuit has two flip-flops A and B, two inputs x and y, and an output z. The flip-flop input functions and the circuit output functions are as follows:

$$J_A = xB + \bar{y}\bar{B}; \quad K_A = x\bar{y}\bar{B}$$

$$J_B = x\bar{A}; \quad K_B = x\bar{y} + A$$

$$z = xyA + \bar{x}\bar{y}\bar{B}$$

Obtain the logic diagram, state table and state equations, also state diagram.

(10 Marks)

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Third Semester B.E. Degree Examination, December 2012
Network Analysis

Time: 3 hrs.

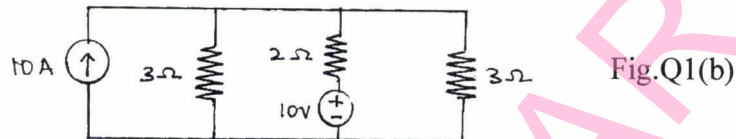
Max. Marks:100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

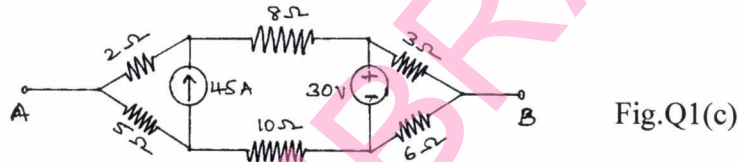
PART – A

- 1 a. Define and distinguish the following network elements:

i) Linear and non-linear	ii) Active and passive	
iii) Lumped and distributed	iv) Ideal and practical current sources	(08 Marks)
- b. Write the mesh equation for the circuit shown in Fig.Q1(b) and determine mesh currents using mesh account analysis. **(06 Marks)**

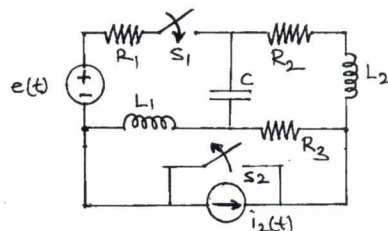
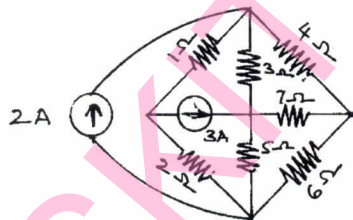


- c. Reduce the network shown in Fig.Q1(c) to a single voltage source in series with a resistance using source shift and source transformations. **(06 Marks)**



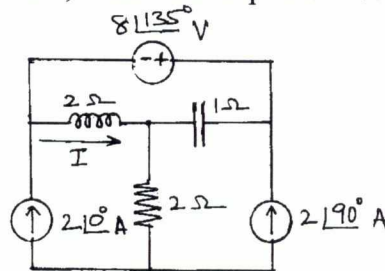
- 2 a. Define the following terms with reference to network topology. Give examples.

i) Tree	ii) Graph	iii) Sub-graph	iv) Tie-set	v) Cut-set	(10 Marks)
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- b. Construct a tree for the network shown in Fig.Q2(b) so that all loop currents pass through 7Ω. Write the corresponding the set matrix. **(06 Marks)**



- c. What are dual networks? Draw the dual of the circuit shown in Fig.Q2(c). **(04 Marks)**

- 3 a. Using superposition theorem, obtain the response I for the network shown in Fig.Q3(a).



(08 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

- 6 b. In the network shown in Fig.Q6(b), 'K' is changed from position 'a' to 'b' at $t = 0$. Solve for i , $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$, if $R = 1000 \Omega$, $L = 1H$ and $C = 0.1\mu F$ and $V = 100V$. Assume that the capacitor is initially uncharged. (10 Marks)

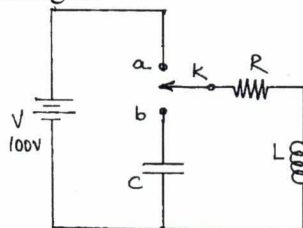


Fig.Q6(b)

- 7 a. Assuming that the staircase waveform of Fig.Q7(a) is not repeated, find its Laplace transform. If this voltage wave is applied to a RL series circuit with $R = 1\Omega$ and $L = 1H$, find the current $i(t)$. (10 Marks)

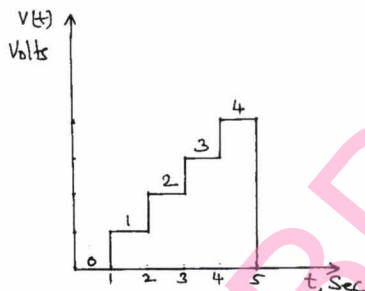


Fig.Q7(a)

- b. The network shown in Fig.Q7(b) was in steady state before $t = 0$. The switch is opened at $t = 0$. Find $i(t)$ for $t > 0$, using Laplace transform. (10 Marks)

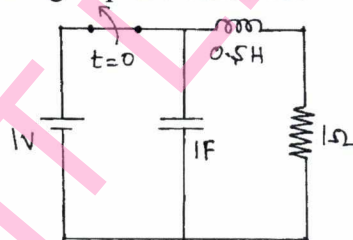


Fig.Q7(b)

- 8 a. Obtain the h-parameters for the network shown in Fig.Q8(a). (10 Marks)

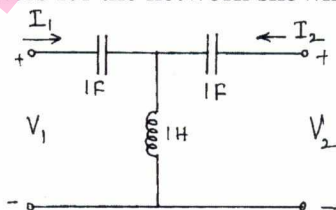


Fig.Q8(a)

- b. Obtain ABCD parameters in terms of z-parameters and hence show that $AD - BC = 1$. (10 Marks)

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Third Semester B.E. Degree Examination, January 2013
Field Theory

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
atleast TWO questions from each part.**

PART – A

1.
 - a. Define ‘Electric field intensity’. Derive an expression for electric field intensity’ (\vec{E}) at a point due to many charges. (07 Marks)
 - b. Point charges of 50 nc each are located at A(1, 0, 0) B(-1, 0, 0) C(0, 10) and D(0, -1, 0)m, find the total force on the charge at A and also find \vec{E} at A. (05 Marks)
 - c. Given $\vec{D} = 5a\vec{r}$ c/m², prove divergence theorem for a shell region enclosed by spherical surfaces at $r = a$ and $r = b$ ($b > a$) and centred at the origin. (08 Marks)

2.
 - a. Find the electric field intensity at point x(1, 2, -1) given the potential $V = 3x^2y + 2y^2z + 3xyz$. (05 Marks)
 - b. Derive boundary conditions between conductor and free space if different ‘ ϵ ’. (08 Marks)
 - c. Show that capacitance of co-axial cable is $C = \frac{2\pi \epsilon L}{\ln[b/a]}$ F with usual notations. (07 Marks)

3.
 - a. With usual representations derive Poisson’s equation. (05 Marks)
 - b. Verify that the potential field given below satisfies the Laplace’s equation $V = 2x^2 - 3y^2 + z^2$. (05 Marks)
 - c. A large spherical cloud of radius ‘b’ has a uniform volume charge distribution of ρ_v c/m³, find the potential distribution and electric field intensity at any point in space using Laplace. (10 Marks)

4.
 - a. State and explain Biot – Savart law. (06 Marks)
 - b. Calculate the value of vector current density in cylindrical co –ordinates at p(1.5, 90°, 0.5) if
$$\vec{H} = \frac{2}{\rho} \cos 0.2\phi \vec{a}_\phi$$
 (06 Marks)
 - c. Given $\vec{H} = 20r^2 \vec{a}_\phi$ A/m, determine the current density J also determine the total current that crosses the surface $r = 1$ m, $0 < \phi < 2\pi$ and $z = 0$ in cylindrical co-ordinate. (08 Marks)

PART – B

5.
 - a. Derive lorentz force equation. (05 Marks)
 - b. Find the force per meter length between two long parallel wires separated by 10 cm in air and carrying a current of 10A in the same direction. (05 Marks)
 - c. Calculate the inductance of a solenoid of 200 turns wound tightly on a cylindrical tube of 6 cm diameter. The length of the tube is 60 cm, the solenoid is in air. Derive the equation for ‘L’. (10 Marks)

- 6 a. Explain Maxwell's equations for time varying fields. (10 Marks)
 b. Find amplitude of displacement current density (J_D) in the free space within a large power distribution transformer $\vec{H} = 10^6 \cos(377t + 1.2566 \times 10^6 z) \vec{a}_y$ A/m. (05 Marks)
 c. Given $H = H_m e^{j(\omega t + \beta z)} \vec{a}_x$ A/m in free space find \vec{E} . (05 Marks)
- 7 a. Starting from Maxwell's equations obtain the general wave equations in electric and magnetic field. (10 Marks)
 b. A 300 MHz uniform plane wave propagates through fresh water for which $\sigma = 0$, $\mu_r = 1$, $\epsilon_r = 78$, calculate :
 i) Attenuation constant
 ii) Phase constant
 iii) Wave length
 iv) Intrinsic impedance. (05 Marks)
 c. State and explain Poynting theorem. (05 Marks)
- 8 a. Define transmission co-efficient and reflection co-efficient deduce the relationship between them. (06 Marks)
 b. A traveling \vec{E} field in the free space of amplitude 100 v/m strikes a perfect dielectric as shown in Fig. Q8(b). Determine E_t . (10 Marks)

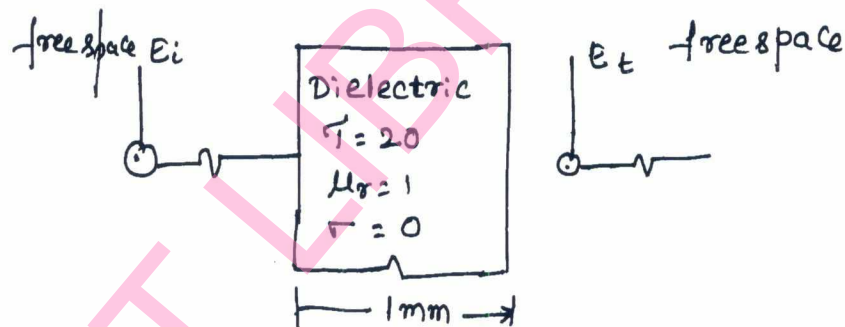


Fig. Q8(b)

- c. Write a note on SWR. (04 Marks)

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Third Semester B.E. Degree Examination, December 2012

Advanced Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer FIVE full questions.

- 1** a. Find the modulus and amplitude of the complex number $1 - \cos \alpha + i \sin \alpha$. (05 Marks)
- b. If z_1 and z_2 are two complex numbers, show that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\{|z_1|^2 + |z_2|^2\}$. (05 Marks)
- c. Find the fourth roots of $-1 + i\sqrt{3}$. (05 Marks)
- d. If $2 \cos \theta = x + \frac{1}{x}$, prove that $2 \cos r\theta = x^r + \frac{1}{x^r}$. (05 Marks)
- 2** a. Find the n^{th} derivative of $e^{2x} \cos^3 x$. (07 Marks)
- b. Find the n^{th} derivative of $\frac{x}{x^2 - 5x + 6}$. (06 Marks)
- c. If $y = e^{a \sin^{-1} x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. (07 Marks)
- 3** a. Find the angle between the pair of curves $r = 6 \cos \theta$, $r = 2(1 + \cos \theta)$. (07 Marks)
- b. Find the pedal equation of the curve $r^2 = a^2 \sin 2\theta$. (06 Marks)
- c. Obtain the Maclaurin's series expansion of the function $\sqrt{1 + \sin 2x}$. (07 Marks)
- 4** a. If $u = x^2y + y^2z + z^2x$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x + y + z)^2$. (05 Marks)
- b. If $u = \tan^{-1}\left(\frac{x^3y^3}{x^3 + y^3}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{3}{2} \sin 2u$. (05 Marks)
- c. If $u = x + y + z$, $v = y + z$, $z = uvw$, find Jacobian of x, y, z with respect to u, v, w . (05 Marks)
- d. If $z = f(x, y)$ and $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$, prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$. (05 Marks)
- 5** a. Obtain the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$ and hence evaluate $\int_0^{\pi/2} \cos^6 x \, dx$ and $\int_0^{\pi/2} \cos^9 x \, dx$. (07 Marks)
- b. Evaluate $\int_0^1 \int_{x^2}^{\sqrt{x}} xy(x + y) \, dy \, dx$. (06 Marks)
- c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dz \, dy \, dx$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8=50, will be treated as malpractice.

- 6 a. Define Gamma and Beta functions. Show that $\beta(m, n) = 2 \int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta \, d\theta$. (07 Marks)
- b. Prove that $\int_0^{\infty} x^2 e^{-x^4} \, dx \times \int_0^{\infty} e^{-x^4} \, dx = \frac{\pi}{8\sqrt{2}}$. (07 Marks)
- c. Evaluate $\int_0^1 (\log x)^6 \, dx$. (06 Marks)
- 7 a. Solve the equation $\frac{dy}{dx} + x \tan(y - x) = 1$. (06 Marks)
- b. Solve $x^2 y \, dx - (x^3 + y^3) \, dy = 0$. (07 Marks)
- c. Solve $(e^y + y \cos xy) \, dx + (x e^y + x \cos xy) \, dy = 0$. (07 Marks)
- 8 a. Solve the equation $(D^3 + 1)y = 0$, where $D = \frac{d}{dx}$. (06 Marks)
- b. Solve the equation $(D^2 - 2D + 1)y = x e^x$. (07 Marks)
- c. Solve $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = e^{2x} - \cos^2 x$. (07 Marks)
